ONSET OF FLUIDIZATION IN A CONICAL SYSTEM

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The onset of instability is considered for a granular bed in a plane-parallel or conical system, and the appropriate critical parameters are calculated.

A granular bed in a container of variable cross section becomes fluidized under certain conditions, and the critical pressure difference is of some interest because this differs considerably from the value for a bed of constant cross section. Particular practical interest attaches to devices converging downward [1, 2]. Here we describe a simple model for the onset of fluidization when the angle of the cone is not too large, with the result that the radial gas flux within a layer can be considered as very largely homogeneous. The main assumptions are very largely derived from those of [3] for granular media flowing through conical bunkers.

Figure 1 shows the bed; we assume that the gas flow in the bed is radially symmetrical, while the lower and upper boundaries are described by R_0 =const, R = const, respectively. These assumptions are closely related and greatly simplify the analysis without altering the physical essence of the matter. In principle, it would be simple to consider a layer with planar boundaries, as has been done for granular media in conical bunkers [4, 5], but in that case the assumption of radial symmetry would become internally contradictory.

For simplicity, we restrict consideration to the stresses in the bed in a simplified one-dimensional setting, as used in deriving Jansen's formula. We assume that the transverse normal stress σ_θ is proportional to the radial stress $\sigma_r = \sigma$, the coefficient of proportionality being $\varkappa$ and characterizing the layer packing. We also assume that there is no adhesion between the particles and no residual stress that does not vanish when the force of gravity tends to zero. These factors [6, 7] can influence the fluidization conditions considerably in an apparatus with vertical walls, but in the present instance they may be relatively unimportant by comparison with the effects of variation in the cross section and the retaining effect of the walls.

Consider the equilibrium of an element (r, r + dr) in a planar layer; the condition for balance between the forces given

$$\theta\{r\sigma(r)-(r+dr)\sigma(r+dr)+[\varkappa\sigma(r)+F(q,\ r)r]\,dr\}-2\sin(\theta/2)\gamma r dr\pm 2k\varkappa\sigma(r)\,dr=0. \tag{1}$$

Here it is assumed that the limiting friction is attained at the walls; the upper and lower signs refer, respectively, to situations where the bed sinks downward or conversely is impelled upward by the hydraulic forces. We assume θ small, so $2\sin(\theta/2) \approx \theta$ and $2k/\theta \gg 1$, and (1) gives

$$\frac{d(r\sigma)}{dr}\mp m\sigma\approx r[F(q,\ r)-\gamma],\quad m=\varkappa\left(\frac{2k}{\theta}\pm 1\right)\approx\frac{2k\varkappa}{\theta}. \tag{2}$$

We use a two-term Erland formula [8] to represent F(q, r), i.e.,

$$F(q,\ r)=\alpha\frac{q}{r}+\beta\left(\frac{q}{r}\right)^2,\quad \alpha=\frac{75}{2}\cdot\frac{(1-\varepsilon)^2}{\varepsilon^3}\cdot\frac{\mu_0}{a^2},\quad \beta=\frac{3{,}5}{4}\cdot\frac{1-\varepsilon}{\varepsilon^3}\cdot\frac{d_0}{a}. \tag{3}$$

The flow rate q is reasonably small, so the force of gravity produces most of the state of stress in the bed; then the upper sign is taken in (2), and the obvious boundary condition is $\sigma(R) = 0$; the solution to (2) then takes the form

$$\sigma=\frac{\gamma}{m-2}r\left(1-\frac{r^{m-2}}{R^{m-2}}\right)-\frac{\alpha q}{m-1}\left(1-\frac{r^{m-1}}{R^{m-1}}\right)-\frac{\beta q^2}{mr}\left(1-\frac{r^m}{R^m}\right). \tag{4}$$

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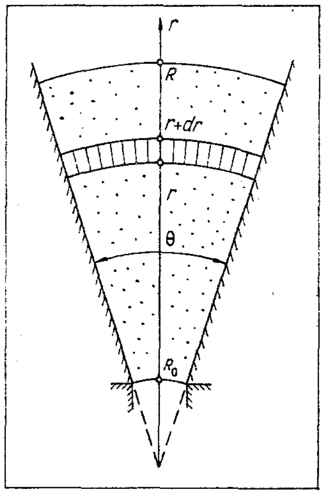

Fig. 1. The apparatus.

This describes the stress distribution in the bed when the gravitational forces predominate, and there is a maximum stress at some critical value of the coordinate, which is dependent on q [3-5]. Clearly (4) is meaningful only for $q < q_1$, where the critical value q_1 is determined by the condition that the stress at the lower boundary of the layer becomes zero. The infiltration speed $u = q/R$ at the upper boundary is introduced, and the corresponding critical value u_1 is then given by

$$u_1^2 + \xi t_0 \frac{m}{m-1} \cdot \frac{1-t_0^{m-1}}{1-t_0^m} u_1 - \eta t_0^2 \frac{m}{m-2} \cdot \frac{1-t_0^{m-2}}{1-t_0^m} = 0, \tag{5}$$

where the symbols are

$$\xi = \frac{\alpha}{\beta} = \frac{150}{3,5} \cdot \frac{(1-\varepsilon)\nu_0}{a}, \quad \eta = \frac{\gamma}{\beta} = \frac{4\varepsilon^3}{3,5} \cdot \frac{d_1}{d_0} ag, \; t_0 = \frac{R_0}{R}. \tag{6}$$

The value q_1/R_0 corresponding to u_1 at the lower boundary exceeds the minimum fluidization speed u_* defined by

$$u_*^2 + \xi u_* - \eta = 0. \tag{7}$$

Consequently, there is a region below the layer in which the hydraulic forces exceed the weight of the particles if $q = q_1$.

If $q > q_1$, the solution of (4) ceases to be correct for some region $R_0 < r < r_*(q)$; if as before we assume that limiting friction applies at the walls, then the stress for this region can be derived by solving (2) with the lower sign in front of the last term on the left and with the boundary condition $\sigma(R_0) = 0$, which gives

$$\sigma = -\frac{\gamma}{m+2} r \left(1 - \frac{R_0^{m+2}}{r^{m+2}}\right) + \frac{\alpha q}{m+1}\left(1 - \frac{R_0^{m+1}}{r^{m+1}}\right) + \frac{\beta q^2}{mr}\left(1 - \frac{R_0^m}{r^m}\right). \tag{8}$$

The critical value of the coordinate $r_*(q)$ is then calculated from the condition of equality of the stresses defined by (4) and (8), which results in the following equation:

$$\gamma r_* \left[\frac{1}{m-2}\left(1 - \frac{r_*^{m-2}}{R^{m-2}}\right) + \frac{1}{m+2}\left(1 - \frac{R_0^{m+2}}{r_*^{m+2}}\right)\right] =$$
$$= \alpha q \left[\frac{1}{m-1}\left(1 - \frac{r_*^{m-1}}{R^{m-1}}\right) + \frac{1}{m+1}\left(1 - \frac{R_0^{m+1}}{r_*^{m+1}}\right)\right] + \frac{\beta q^2}{m r_*}\left(2 - \frac{r_*^m}{R^m} - \frac{R_0^m}{r_*^m}\right). \tag{9}$$

The range of application of (8) increases monotonically with q until the upper boundary reaches the upper edge of the layer at some critical flow rate $q = q_2 > q_1$; the corresponding critical speed $u_2 = q_2/R$ at the upper boundary satisfies

$$u_2^2 + \xi \frac{m}{m+1} \cdot \frac{1-t_0^{m+1}}{1-t_0^m} u_2 - \eta \frac{m}{m+2} \cdot \frac{1-t_0^{m+2}}{1-t_0^m} = 0. \tag{10}$$

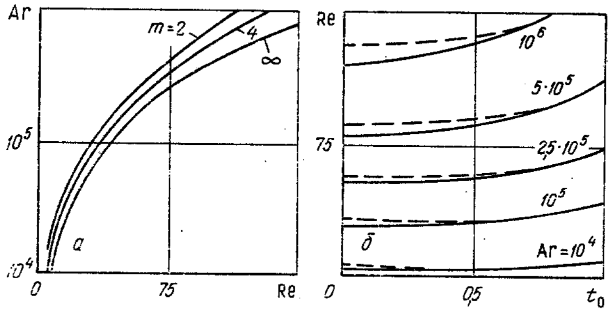

Fig. 2. a) Critical Reynolds number corresponding to loss of stability as a function of the Archimedes number Ar for $t_0 \approx 0$ and various m: b) Re as a function for t_0 for various Ar for m = 2 (solid lines) and m = 3 (dashed lines).

The stresses are everywhere defined by (8) if $q = q_2$, and they become zero at both boundaries of the bed; it is clear that no equilibrium state exists for the bed if $q > q_2$ (or $u > u_2$), so u_2 may be taken as representing onset of fluidization. Note that this value is less than the minimum fluidization speed defined by (7).

It is convenient to put (10) in standard form by introducing the Reynolds and Archimedes numbers as follows:

$$\mathrm{Re} = \frac{2au_2}{\nu_0}, \quad \mathrm{Ar} = \frac{8a^3}{\nu_0^2} \cdot \frac{d_1}{d_0} g. \tag{11}$$

Then simple transformation via (3) and (6) converts (10) for $\varepsilon = 0.4$ to

$$\mathrm{Re}^2 + 51.4 \frac{m}{m+1} \cdot \frac{1 - t_0^{m+1}}{1 - t_0^m} \mathrm{Re} - 0.0366 \frac{m}{m+2} \cdot \frac{1 - t_0^{m+2}}{1 - t_0^m} \mathrm{Ar} = 0. \tag{12}$$

This equation simplifies somewhat in the important particular case $t_0 \ll 1$ provided that $\theta \to 0$ ($m \to \infty$), i.e., for an equipment with vertical walls, whereupon (12) takes a standard form [9]:

$$\mathrm{Re}^2 + 51.4\mathrm{Re} + 0.0366\mathrm{Ar} = 0. \tag{13}$$

We process (12) as for (13) in [10] to get a simplified formula:

$$\mathrm{Re} = \frac{\mathrm{Ar}}{1400 \dfrac{m+2}{m+1} \cdot \dfrac{1 - t_0^{m+1}}{1 - t_0^{m+2}} + 5.22 \left(\dfrac{m+2}{m} \cdot \dfrac{1 - t_0^m}{1 - t_0^{m+2}} \mathrm{Ar} \right)^{1/2}}. \tag{14}$$

This formula coincides with that derived in [10] for $m \to \infty$; the relationship between the critical Re and Ar implied by (12) is shown in Fig. 2a for various m for the limiting case $t_0 \to 0$. Figure 2b shows Re as a function of t_0 for two values of m and various Ar.

The model of [11] implies that the fluidized state should occur when q_1 is reached, because then a point appears where the normal stress is zero. In fact, the transition extends up to the second critical flow rate q_2, which corresponds to stability loss in the bed as a whole, and this value may exceed q_1 substantially, particularly for t_0 small.

The following is the pressure difference across the bed for $q < q_2$:

$$\Delta p = \int_{R_0}^{R} \left(\frac{\alpha q}{r} + \frac{\beta q^2}{r^2} \right) dr = \alpha q \ln \frac{R}{R_0} + \beta q^2 \left(\frac{1}{R_0} - \frac{1}{R} \right). \tag{15}$$

The critical pressure difference for stability loss is then derived for $q = q_2$, the result being very much dependent on the size R_0 of the base of the bed; the latter is reflected in empirical formulas for Δp, as in [1, 2].

From (15) we readily get simplified formulas that apply approximately in various particular situations. For instance, if the particles are not too small, with the result that the second term in (3) is no less important than the first, we have that the second term in (15) plays the main part for $R_0 \ll R$, and the height of the bed is $H \approx R$. The width of the base of the bed is $L \approx 2R_0\tan(\theta/2)$, which gives us the following formula for the critical pressure difference:

$$\Delta p \approx [\beta u_*^2 H] \left(2H \operatorname{tg} \frac{\theta}{2} \Big/ L\right) \tag{16}$$

(u_2 has here been replaced by u_*, which involves a small error), and the cofactor in the brackets is the critical pressure difference for a bed with vertical boundaries. A formula of the same type has been suggested elsewhere [2].

In the above we have considered an element of a bed in a triangular bunker with inclined walls; all the results are however readily extended to a bed in an axially symmetrical cone. If q denotes the flow in unit solid angle, the above approximations for σ give us instead of (2) that

$$\frac{d(r^2\sigma)}{dr} \mp 2mr\sigma = r^2\left[\alpha\frac{q}{r^2} + \beta\frac{q^2}{r^4} - \gamma\right], \tag{17}$$

with m still defined by (2), while θ is now the angle of the cone.

The analogs of (4) and (8) are correspondingly

$$\sigma = \frac{\gamma r}{2m-3}\left(1-\frac{r^{2m-3}}{R^{2m-3}}\right) - \frac{\alpha q}{(2m-1)r}\left(1-\frac{r^{2m-1}}{R^{2m-1}}\right) - \frac{\beta q^2}{(2m+1)r^3}\left(1-\frac{r^{2m+1}}{R^{2m+1}}\right), \tag{18}$$

$$\sigma = -\frac{\gamma r}{2m+3}\left(1-\frac{R_0^{2m+3}}{r^{2m+3}}\right) + \frac{\alpha q}{(2m+1)r}\left(1-\frac{R_0^{2m+1}}{r^{2m+1}}\right) + \frac{\beta q^2}{(2m-1)r^3}\left(1-\frac{R_0^{2m-1}}{r^{2m-1}}\right). \tag{19}$$

The linear velocity at the upper boundary of the bed is now defined by $u = q/R^2$, while the critical value u_2 is obtained subject to the condition that (19) becomes zero for $r = R$. We use (11) to get instead of (12) in that case that

$$\mathrm{Re}^2 + 51.4\,\frac{2m-1}{2m+1}\cdot\frac{1-t_0^{2m+1}}{1-t_0^{2m-1}}\,\mathrm{Re} - 0.0366\,\frac{2m-1}{2m+3}\,\frac{1-t_0^{2m+3}}{1-t_0^{2m-1}}\,\mathrm{Ar} = 0. \tag{20}$$

This expression again becomes (13) for $m \to \infty$, which applies for an equipment with vertical walls. A general form of the relationship of Re to t_0 and Ar corresponding to (20) is as shown in Fig. 2.

The pressure difference across the axially symmetrical layer is

$$\Delta p = \int_{R_0}^{R}\left(\frac{\alpha q}{r^2} + \frac{\beta q^2}{r^4}\right)dr = \alpha q\left(\frac{1}{R_0} - \frac{1}{R}\right) + \frac{\beta q^2}{3}\left(\frac{1}{R_0^3} - \frac{1}{R^3}\right). \tag{21}$$

A formula of the type of (16) is obtained for a bed of reasonably small particles if $\beta q \ll \alpha R_0^2$ and $R_0 \ll R$.

If we neglect the frictional forces at the wall, then the result for a granular bed in a conical system has been derived previously [12], where the critical flow speed corresponding to loss of stability was defined from the condition for balancing the weight of the bed (as corrected for the upthrust). It is clear that the situation with vanishingly small friction is described by the above formulas if we take the particular case $m = 0$. The equation for the Reynolds number corresponding to the onset of fluidization derived from (20) with $m = 0$ coincides with the equation derived from [12] for θ small. Figure 2a shows that any increase in the friction at the wall tends to increase this number, so the value calculated without correction for the friction must be somewhat too low. This is confirmed to some extent by the data of [13], in which measurements were compared with calculations from various sources.

The state of the bed after stability loss (i.e., for $q > q_2$) cannot be described within the framework of this theory; however, it is possible to have two limiting situations if q is not too greatly in excess of the critical value, and these we discuss briefly. First of all, the layer as a whole may remain immobile but shift upward by an amount such that the new boundaries are described by the coordinates $R_0' > R_0$ and $R' > R$, with the condition for constancy of volume of the bed giving $R'^2 - R_0'^2 = R^2 - R_0^2$ if we neglect any possible change in the proportion of voids. The second equation needed to determine R' and R_0' as a function of q is derived by setting the stress at the upper boundary $r = R'$ equal to zero. This stress at the upper boundary is still described by (8) or (19) with $q > q_2$, but with R_0 replaced by R_0'.

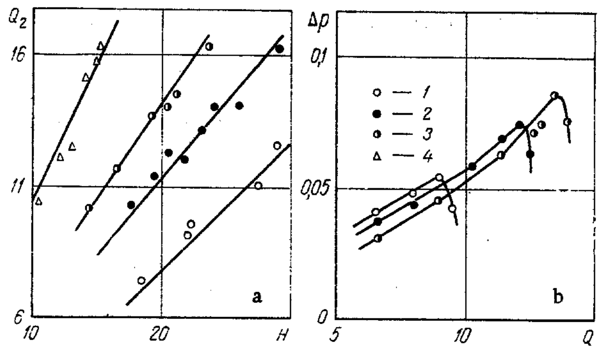

Fig. 3. a) Critical flow rate $Q_2 = \theta q_2$ (m^3/h) as a function of height of granular bed H (cm) for various θ; b) pressure difference Δp (kgf/cm^2) as a function of flow rate $Q = \theta q$ (m^3/h) for various θ. The solid lines are from theory, while the points are from experiment for θ as follows: 1) 10°; 2) 15; 3) 20; the points 4 represent a gas jet breaking through the immobile bed.

The behavior of this equation for realistic m indicates that $t = R_0'/R'$ increases rapidly from the small initial value t_0 as q increases, particularly over a narrow range immediately to the right of q_2; correspondingly, there is a rapid increase in R_0', with the result that the pressure difference is still defined by (15) or (21), although for $q > q_2$ the result is a very rapidly decreasing function of q.

The existence of this state requires that the lower surface is stable against fall of individual particles; it is clear [14] that this requires that the relative velocity of the gas is on the order of the entrainment speed (is less than the latter by not more than a factor 2-3). It is clear that this condition is violated as R_0' increases, so the lower surface becomes unstable, with the consequence that the lower part of the bed or even the entire bed becomes essentially fluidized. It is clear that this stage can be attained if q is increased sufficiently slowly provided that the coefficient of dynamic friction at the wall is not much less than the static value. If these conditions are not met, the bed suddenly jumps upward when the point $q = q_2$ is reached, and random perturbations that prevent the formation of a sharp lower boundary may become very strong. In that case, the second limiting state is more probable, in which the upper part of the bed is immobile, while the lower part is fluidized. The stability is then lost when q increases further, with the result that the entire bed becomes fluidized or a jet system is set up within the apparatus.

The above general picture is confirmed by trials on fluidization of a granular bed in a trough with sloping sides. The bed consisted of polystyrene particles of diameter 2.5 mm, while θ took the values 10, 15, 20, and 30°. The air was injected through a slot of width 2 mm, while the length of the trough was 100 mm; the height of the bed varied within wide limits. The critical flow rate θq_2 and the pressure differences across the bed before and after onset of instability were determined. Figure 3 compares the theoretical results with the measurements. The calculations were based on $\varepsilon = 0.4$, while $k\varkappa$ was determined from a single experiment. Figure 3 shows satisfactory agreement.

If the flow rate is gradually increased above the critical value, the layer of material rises as a solid body without any appreciable signs of loss of continuity; the lower boundary is at first stable, with just a few particles circulating in the space under the bed. A further increase in the flow rate produces a new upward shift, with some partial loss of stability in the lower boundary (more circulating particles), and the proportion of these continues to increase with q until there is no longer a clear-cut division between the fluidized and unfluidized parts. Channels can arise before the value $q_* = u_* R$ is reached, which may penetrate to the upper part, and periodic bubbles break through to the upper surface as q is increased further, after which a steady-state fountain sets in. Figure 4 shows characteristic pictures reflecting the state of the bed. These various types of fluidization have also been reported elsewhere [13].

These considerations can also be used in approximate analysis of a horizontally unbounded bed injected with a gas jet; the gas enters through a slot or hole into the base of the bed, which is accompanied by the formation of cavities containing rapidly circulating particles, whose size increases with the gas flow rate.

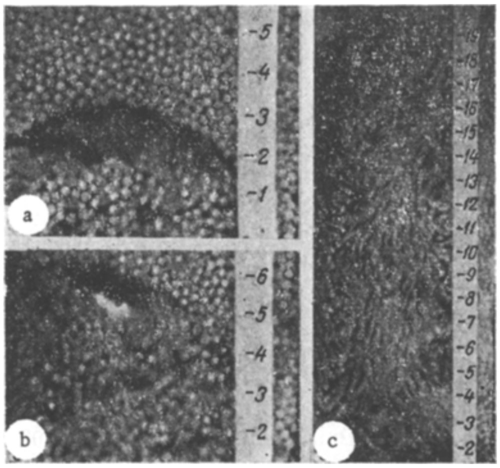

Fig. 4. The characteristic states of a granular bed, photographs arranged in order of increasing flow rate.

When these cavities attain about 70% of the height of the bed, there is a tendency for them to break through to the surface and produce a stable fountain. The state before breakdown is one with the bed distorted, with a convex area on the surface of the bed.

Most of the gas passes through the bed near the walls, and the deformed area on the surface corresponds to the base of the bed. There is thus a clear analogy between the above cavities and the space under the immobile part of the bed in a conical or through system, and the above equations can be applied as an approximation. This approach is also justified by a comparison of the observed and theoretical relationships for the critical flow rate corresponding to breakthrough of a jet from a slot (Fig. 3a). The first curve was derived in experiments with beds of the above polystyrene particles; the second was calculated from the above formulas with the observed θ of 25°, while the coefficient of friction k was taken as parameter. Here again the agreement between theory and experiment was satisfactory.

NOTATION

a	is the particle radius;
d_0, d_1	are the density of gas and particle material;
F	is the hydraulic force;
g	is the acceleration due to gravity;
H	is the height of bed;
k	is the wall friction coefficient;
L	is the transverse size of lower boundary;
m	is the parameter in (2);
Δp	is the pressure drop;
q	is the gas flow rate per unit planar or solid angle;
R_0, R	are the coordinates of lower and upper boundaries, respectively;
r_*	is the coordinate for $t_0 = R_0/R$;
u	is the gas flow rate at upper boundary;
u_*	is the minimum fluidization speed;
α, β	are the parameters defined in (3);
γ	is the apparent density of granular bed;
ε	is the porosity;
θ	is the angle between inclined walls or vertex angle of conical apparatus;
$\varkappa$	is the coefficient of proportionality between normal stresses;
μ_0	is the viscosity;
ν_0	is the kinematic viscosity;
ξ, η	are the parameters in (6);
σ	is the normal stress. A prime denotes the raised bed.

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DRYING IN HEATED GAS FLOWS

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A study has been made of the drying of a body by passage of a drying agent; various structures have been used. A simple mathematical model is presented for this type of drying.

Much attention is being given to the drying of gas-permeable bodies by passage of a heat carrier through a planar layer of material [1-4]. Usually, such a material has large internal channels and pores, in which the hydraulic resistance is quite low [2]. It has been found that the drying is then considerably more rapid than convective drying.

Particular interest attaches to research on drying of this type for materials differing considerably in structure and type of water binding. We have examined various materials (felt, cardboard, nonwoven fabrics, sheet asbestos, woven asbestos strip, and the like), which differ in nature and hydraulic resistance. The measurements were made over a wide temperature range with widely varying pressure differences, the main working unit for the purpose being that shown in Fig. 1, which consists of two sections 1 and 2, which are separated by the perforated baffle 3. Leakage around the edge is prevented by the sealing ring 4, which is compressed by the cover 5. The drying is performed as follows. The wet specimen 6 of diameter 100 mm is set up in section 1 on the perforated baffle 3, with a vacuum set up in section 2. The gas at a set temperature is supplied to the surface of the specimen and passes through it as a result of the pressure difference.

Figure 2 shows results for felt, cardboard, and woven asbestos strip at 100°C and a pressure difference of 65,000 N/m^2. The kinetic curves indicate the mode of drying in the different groups of materials, which differ considerably in structure.

The felt had coarse pores with low hydraulic resistance; the woven asbestos strips were much denser and bound the water in a different fashion. Cardboard is a typical colloidal porous material dominated by small capillaries, and it has the highest hydraulic resistance. Curve 1 reflects the drying of felt of thickness 10 mm and has three prominent parts. The first is rapid mechanical displacement of the water by the gas, while the second and third are drying proper. About half of the water is eliminated during the first period. Curve 2 represents the asbestos strip of thickness 10 mm, which takes the form of a classical curve with two periods. Here mechanical displacement plays no definite part. Curve 3 indicates the drying of cardboard of

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